Title: Time-Stepping Methods for PDEs and Ocean Models

Abstract: My doctoral research can be broadly divided into two parts:

a. **On the Spatial and Temporal Order of Convergence of Hyperbolic PDEs:** In this part, I discuss the leading order terms of the local truncation error of hyperbolic partial differential equations (PDEs). If one employs a stable numerical scheme and the global solution error is of the same order of accuracy as the global truncation error, one can make the following observations in the asymptotic regime, where the truncation error is dominated by the powers of grid-spacing and time step rather than their coefficients. Assuming that the spatial and temporal resolutions reach the asymptotic regime before the machine precision error dominates,

    i. the order of convergence of stable numerical solutions of hyperbolic PDEs at constant ratio of time step to grid-spacing is governed by the minimum of the orders of the spatial and temporal discretizations, and

    ii. convergence cannot even be guaranteed under only spatial or temporal refinement.

    I have tested the theory against numerical methods employing Method of Lines and not against ones that treat space and time together. Otherwise, the theory applies to any hyperbolic PDE, be it linear or non-linear, and employing finite difference, finite volume, or finite element discretization in space, and advanced in time with a predictor-corrector, multistep, or a deferred correction method. If the PDE is reduced to an ordinary differential equation (ODE) by specifying all spatial gradients to be zero, then the standard local truncation error of the ODE is recovered. I perform the analysis with generic and specific hyperbolic PDEs using the symbolic algebra package SymPy, and conduct a number of numerical experiments to demonstrate the theoretical findings.

b. **Time-Stepping Methods for Ocean Models:** In the second part of my doctoral research, I study and address some complications associated with time-stepping the prognostic equations of an ocean model. The primary one is the disparate time scales problem, leading to a splitting of the fast depth-independent 2D barotropic modes and the slow 3D baroclinic modes, and the application of a time-averaging filter over the barotropic modes to minimize aliasing and mode-splitting errors. To study the combined stabilizing effect of various filters and the forward-backward parameter, I develop a non-linear shallow water solver, simulate a surface gravity wave, and use the magnitude of the SSH error norm as an indicator of the right amount of dissipation. I redesign parts of the time-stepping algorithm of the Model for Prediction Across Scales — Ocean (MPAS-O) developed at the Los Alamos National Laboratory (LANL), to improve the barotropic-baroclinic splitting and enhance the coupling between the two modes. I incorporate a number of barotropic time-averaging filters in MPAS-O. I repeat the surface gravity wave test case with the non-rotating primitive equations of MPAS-O, and analyze the influence of the filters on the numerical solution. Finally, I design a verification suite of shallow water test cases for the barotropic solver of ocean models. I develop an unstructured-mesh ocean model in object-oriented Python, employing the TRiSK-based spatial discretization of MPAS-O, and numerous time-stepping methods, and use it as the platform to run the shallow water test cases. I conclude my doctoral research by conducting convergence studies for each test case keeping the time step proportional to cell width, and verifying that the convergence rates match the ones predicted by the theory in part (a).