An Edge-Weighted Centroidal Voronoi Tessellation Model For Image Segmentation

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1.Introduction

Centroidal Voronoi Tessellations(CVT's) are special Voronoi Tessellations whose generators are also the centers of mass (centroids) of the Voronoi regions with respect to a given density function. The classic CVT's can efficiently produce a segmentation for homogeneous images but fails to handle the inhomogeneous ones.

In order to overcome some deficiencies, we develop an *edge-weighted centroidal Voronoi Tessellation* (EWCVT) model by appropriately combining the image intensity information together with the length of cluster Boundaries.

2. Classic CVT

Voronoi Regions VS Centrodal Voronoi Tessellation

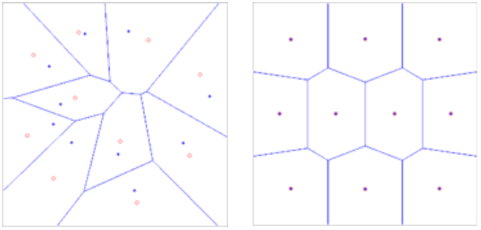

Fig. 1: Left: Voronoi regions and their centers of mass (given a uniform density) for 10 randomly sampling points in a square; note, the generators (the stars) do not coincide with the centers of mass (the circles)

Right: A centroidal Voronoi Tessellations in which the generators of the Voronoi regions are simultaneously the centers of mass (the same uniform density with left figure) of the Voronoi cells

Energy Functional of Classic CVT

Given

- A image $\mathrm{U}=\{u(i,j)\}_{(i,j)\in D}$
- A set of generators $\mathrm{W}=\{w_l\}_{l=1}^{L}$

Voronoi regions are defined by:

$$V_k=\{u(i,j)\in \mathrm{U}:\ |u(i,j)-w_k|\le|u(i,j)-w_l|,\ l=1,\cdots L\}\quad k=1,\cdots L.$$

We need to minimize the classic CVT energy:

$$E_{VT}(\mathrm{W})=E(\mathrm{W};\mathrm{V})=\sum_{l=1}^{L}\sum_{u(i,j)\in V_l}|u(i,j)-w_l|^2$$

The minimizer is the Centroidal Voronoi Tessellations in which $w_k=\overline{w}_k$, i.e., the generators are coincide with the centers of mass of each Voronoi regions.

Deficiencies of Classic CVT

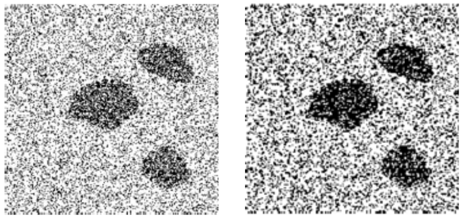

Fig. 2. Left: the original image of a simulated noisy minefield. Right: CVT-based segmentation into two clusters.

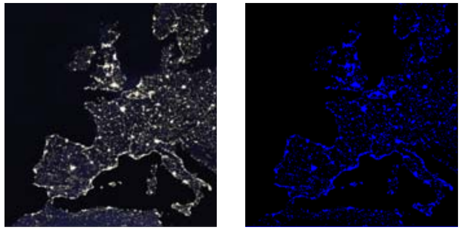

Fig. 3. Left: the original image of "Europe-at-night". Right: CVT-based segmentation into two clusters.

In Figure 1, the CVT-based segmentation failed to accurately identify which objects are mines and which are just noise. In Figure 2, the CVT-based segmentation efficiently identify the lights, but one fails to recognize the boundary between the European landmass and the Atlantic ocean.

3. Edge-Weighted CVT Model

Edge Energy (related to boundaries)

Define local characteristic function $\chi_{(i,j)}:\mathbb{N}_\omega(i,j)\to\{0,1\}$:

$$\chi_{(i,j)}(i',j')=\begin{cases}1, & \text{if } \pi_{\mathrm{U}}(i',j')\neq\pi_{\mathrm{U}}(i,j);\\ 0, & \text{otherwise;}\end{cases}$$

- $\mathbb{N}_\omega(i,j)$ Neighborhood of pixel (i,j)
- $\pi_{\mathrm{U}}(i,j)=l,\ \text{if } (i,j)\in U_l$

The edge energy for each pixel is defined as

$$\varepsilon_L(i,j)=\lambda\sum_{(i',j')}\chi_{(i,j)}(i',j')$$

where λ is a positive weighting factor.

The total edge energy is given by

$$E_L(D)=\sum_{(i,j)\in D}\varepsilon_L(i,j)$$

Energy Functional for Edge-Weighted CVT Model

Together with the classical clustering energy adopted by the classical CVT model, we define the *edge-weighted clustering energy (EWCE)* as follows:

$$\begin{aligned}\hat{E}(\mathrm{W};\mathrm{U})&=E(\mathrm{W};\mathrm{U})+E_L(\mathrm{U})\\&=\sum_{l=1}^{L}\sum_{(i,j)\in U_l}|u(i,j)-w_l|^2\\&\quad+\lambda\sum_{(i,j)\in D}\sum_{(i',j')\in\mathbb{N}_\omega(i,j)}\chi_{(i,j)}(i',j')\end{aligned}$$

Same as before, this is the energy need to be minimized.

Algorithms to minimize the above energy functional

Given an integer L and choose arbitrarily a partition $\{\tilde{D}_l\}_{l=1}^{L}$ of image $\mathrm{U}=\{u(i,j)\}_{(i,j)\in D}$. Determine the centroids $\{w_l\}_{l=1}^{L}$ of $\{\tilde{D}_l\}_{l=1}^{L}$ and take them as generators.

1. For every pixel (i,j), try to move (i,j) to every other clusters, accept the move which results in most energy reduction. Then replace w_l and w_m with the centroids of the newly modified clusters $\tilde{D}_l$ and $\tilde{D}_m$ respectively.
2. if no pixel is moved, return $\{w_l\}_{l=1}^{L}$ and $\{\tilde{D}_l\}_{l=1}^{L}$. Then exit the loop; otherwise, go to step 1.

4. Experiments AND Discussions

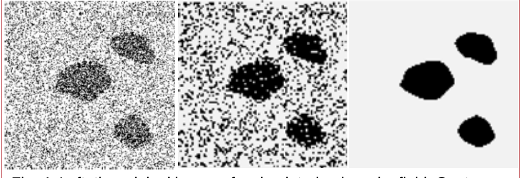

Fig. 4. Left: the original image of a simulated noisy minefield. Center: initial clusters obtained by classic CVT. Right: segmentation obtained by Edge-Weighted CVT.

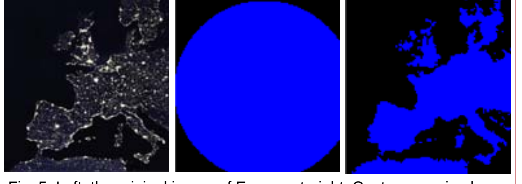

Fig. 5. Left: the original image of Europe-at-night. Center: use simple circle as initial clusters. Right: segmentation obtained by Edge-Weighted CVT.

Figure 4 and 5 demonstrate our algorithms are quite robust with respect to the selection of initial clusters.

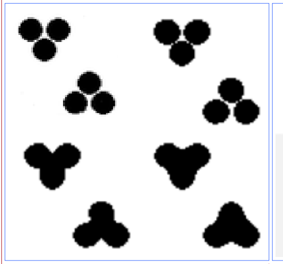

Fig. 6: different segmentations obtained by using different weighting factor λ

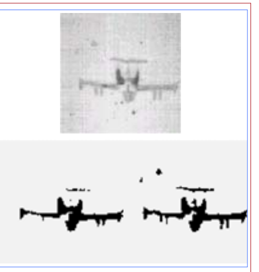

Fig. 7: different segmentations obtained by using different neighborhood size ω

Figure 6 and 7 show the flexibility of our algorithm to meet different needs of various applications.

Our algorithms can produce pretty good segmentations even for quite complex images from real world scenes.

Figure 8: Detection of skiers from a real-world background

Figure 9: Detection of elephants from a savanna scene

5. Conclusion Remarks

- EWCVT is computationally less expensive than PDE based algorithms.
- EWCVT is easy to be generalized to handle any number of clusters
- EWCVT is robustness with respect to noise and flexible to control the segmentation accuracy.

6. Reference

- Jie Wang, Lili Ju and Xiaoqiang Wang, An Edge-Weighted Centroidal Voronoi Tessellation Model For Image Segmentation, accepted by IEEE transaction on Image Processing