Correcting Asymmetric Lens Distortion with Thin Plate Splines

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Abstract

The field of Geometric Morphometrics is concerned with the multivariate statistical analysis of shape, particularly biological shape, often focusing on the coordinates of meaningful landmarks located on specimens. (Adams, et al. 2004) The bulk of geometric morphometric data are collected by digitizing these landmark points from photographs. Thus, it is of general interest to morphometricians that photographs reflect accurately the geometry their subjects. However, due to design and manufacturing imperfections of camera lens elements, coordinates of points in photographs appear displaced from the desired rectilinear projection. Many common methods of correcting lens distortion assume that displacements are symmetric around the center of the photograph, and may not capture the full nature of the distortion. Here, Fred Bookstein's thin plate splines, commonly used by morphometricians for other purposes, are used in an attempt to solve the problem of asymmetric lens distortion. For this project, software was constructed in Java, using the Apache Math Commons linear algebra library for all matrix algebra, and using the Swing GUI toolkit to visualize the nature of the lens distortion.

Background Knowledge

A digital camera operates by projecting light from a scene onto the photo- graphic medium (generally either CCD or CMOS), by means of a photographic lens, rendering a digital representation of a scene. An unfortunate consequence of this process is that error is introduced by the lens, causing coordinates of points in the scene to be displaced in the photograph. Duane C. Brown, in his paper offering an analytic method to correct this type of distortion, stated that "The distortion of a perfectly centered lens composed of individually flawless elements is strictly symmetric about the optical axis." (Brown 1966) However, lenses suffer from design and manufacturing imperfections, which may be of consequence to those who require accurate photographs. (Arnqvist and Martensson 1998) Imagined as the deflection of points on a thin sheet of metal, thin plate splines are used to apply a spatial mapping from one set of points exactly to another, while interpolating the values in between. (Bookstein 1989) By photographing an object with known geometry (here, a piece of paper with regularly-spaced dots) and comparing the resulting photograph with the "ideal" representation, thin plate splines can be used to correct the distortion present in the photograph. Given that the camera configuration remains the same, subsequent photographs can be corrected using the same parameters. The so-called "bending energy" I_f is minimized so that only one solution exists.

$$I_f = \iint_{R^2} \left(\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) dxdy$$

The x and y displacements are computed separately with the function:

$$f(x, y) = a_1 + a_x x + a_y y + \sum_{i=1}^{n} w_i U(|P_i - (x, y)|)$$
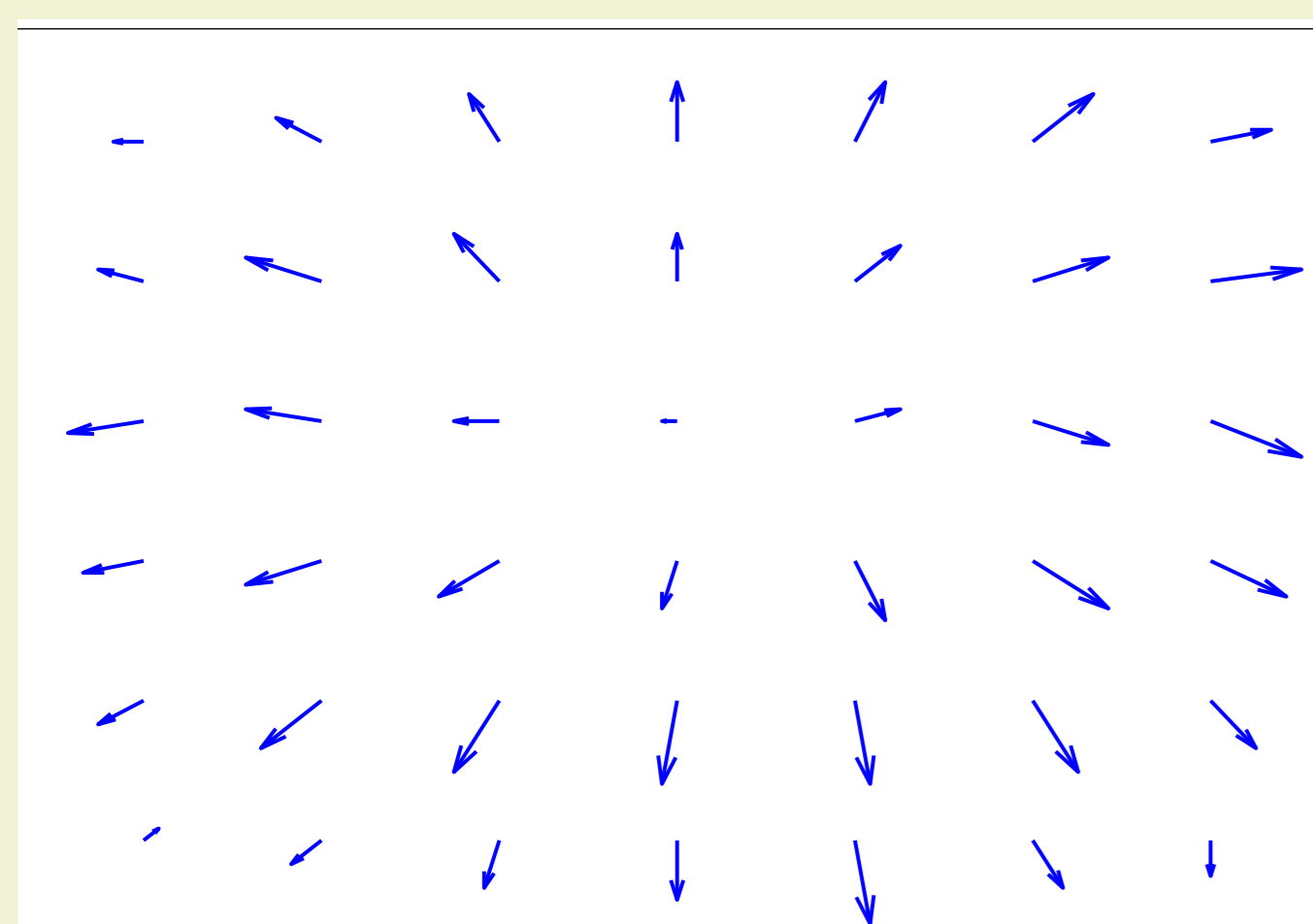
The unknown parameters are determined by solving the linear system

$$\begin{bmatrix} K & P \\ P^T & O \end{bmatrix} \begin{bmatrix} \vec{w} \\ \vec{a} \end{bmatrix} = \begin{bmatrix} \vec{v} \\ \vec{o} \end{bmatrix}, \text{ where } K = \begin{bmatrix} 0 & U(r_{21}) & \dots & U(r_{1n}) \\ U(r_{21}) & 0 & \cdots & U(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ U(r_{n1}) & U(r_{n2}) & \cdots & 0 \end{bmatrix},$$
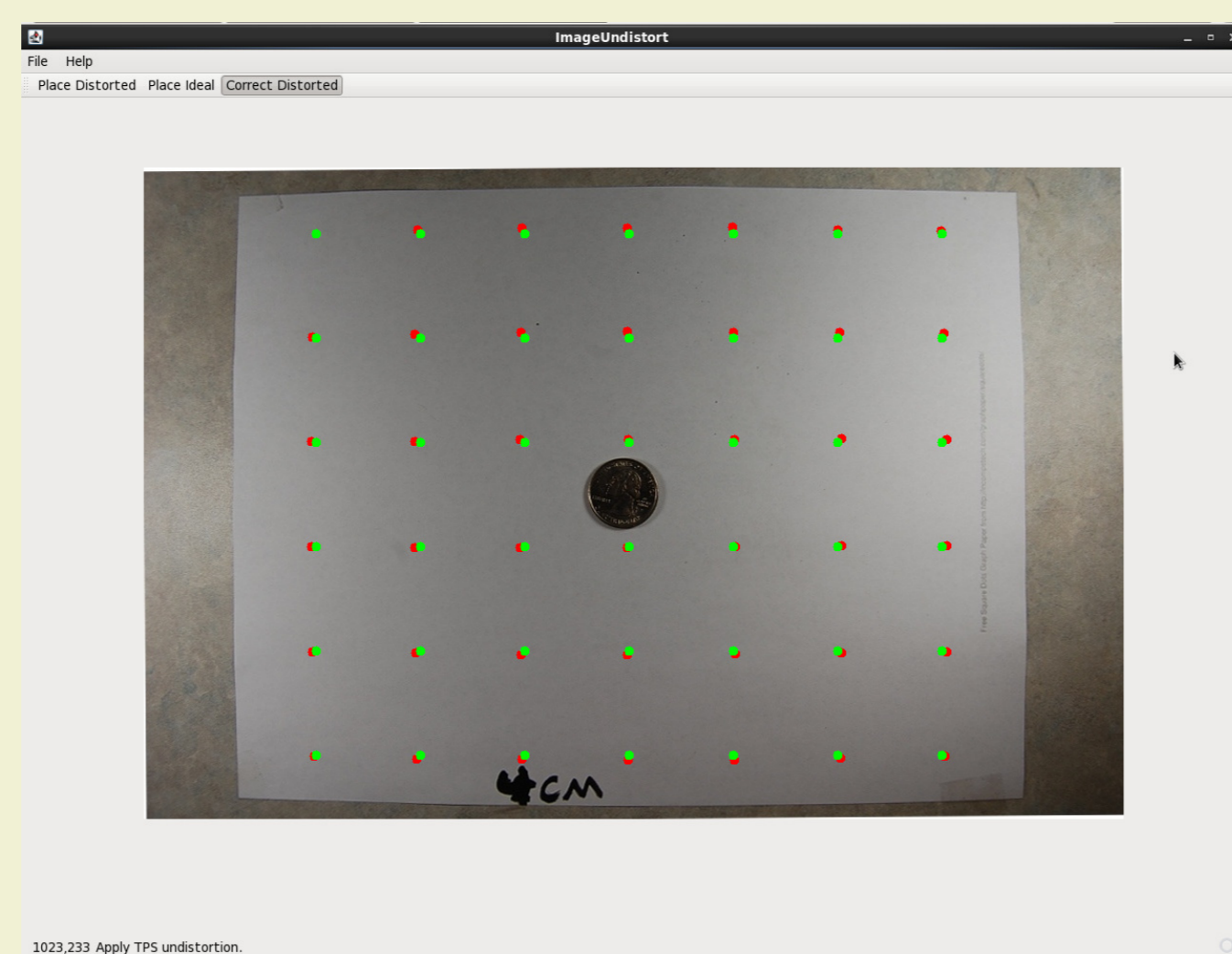
$$P = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}, \ P_1 = (x_1, y_1), \cdots, P_n = (x_n, y_n), \ r_{ij} = |P_i - P_j|,$$

$U(r)$ is the radial basis function $-r^2 log(r^2)$, and O is a 3x3 matrix of zeros.

Screenshot and Scaled Distortion Map

(a) Point Displacement Map (scaled)   (b) Screenshot of Software

## References

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Arnqvist, G., and T. Martensson. 1998. Measurement error in geometric morphometrics: Empirical strategies to assess and reduce its impact on measures of shape. Acta Zoologica Academiae Scientiarum Hungaricae 44: 73-96.

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Distorted vs. Corrected Image
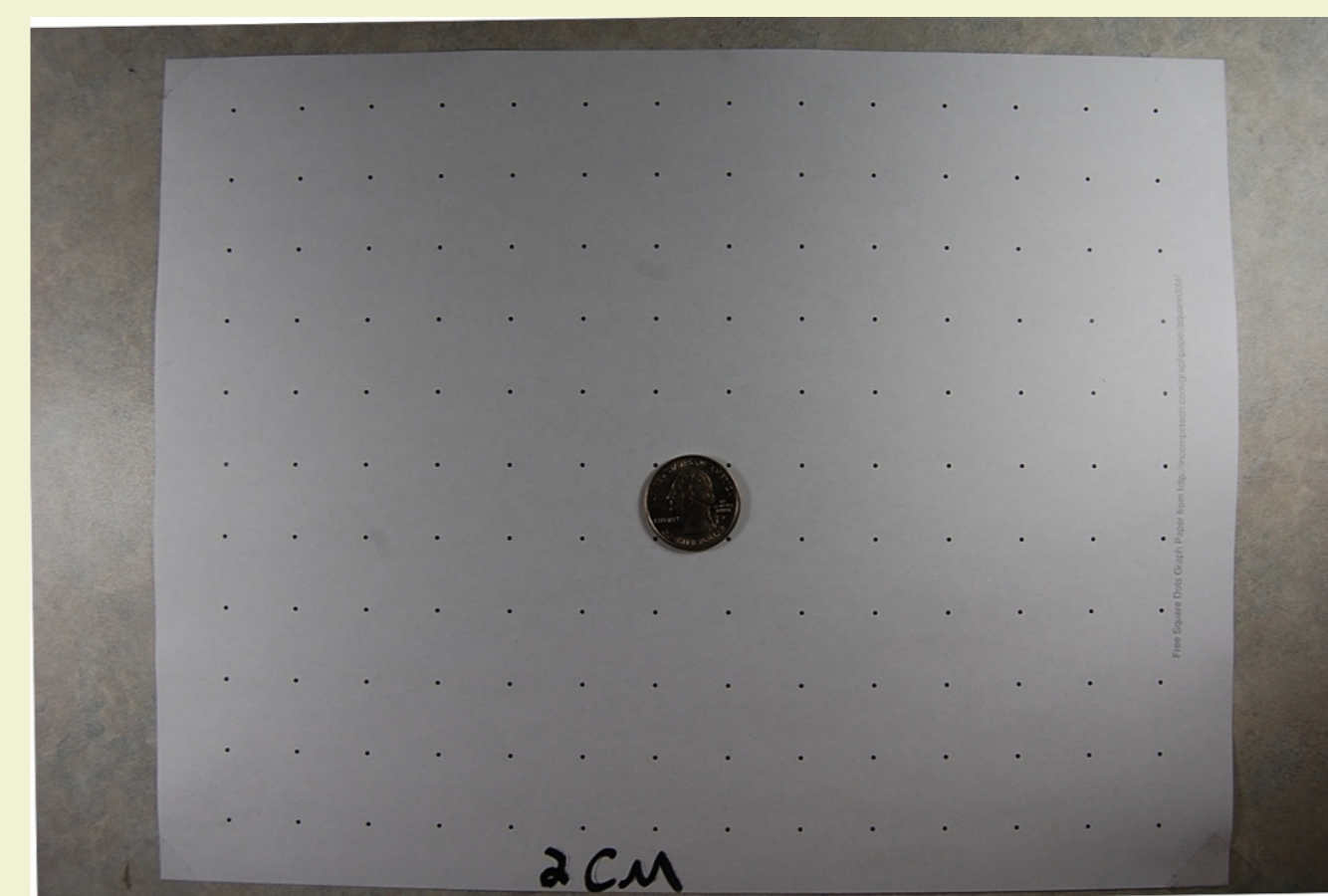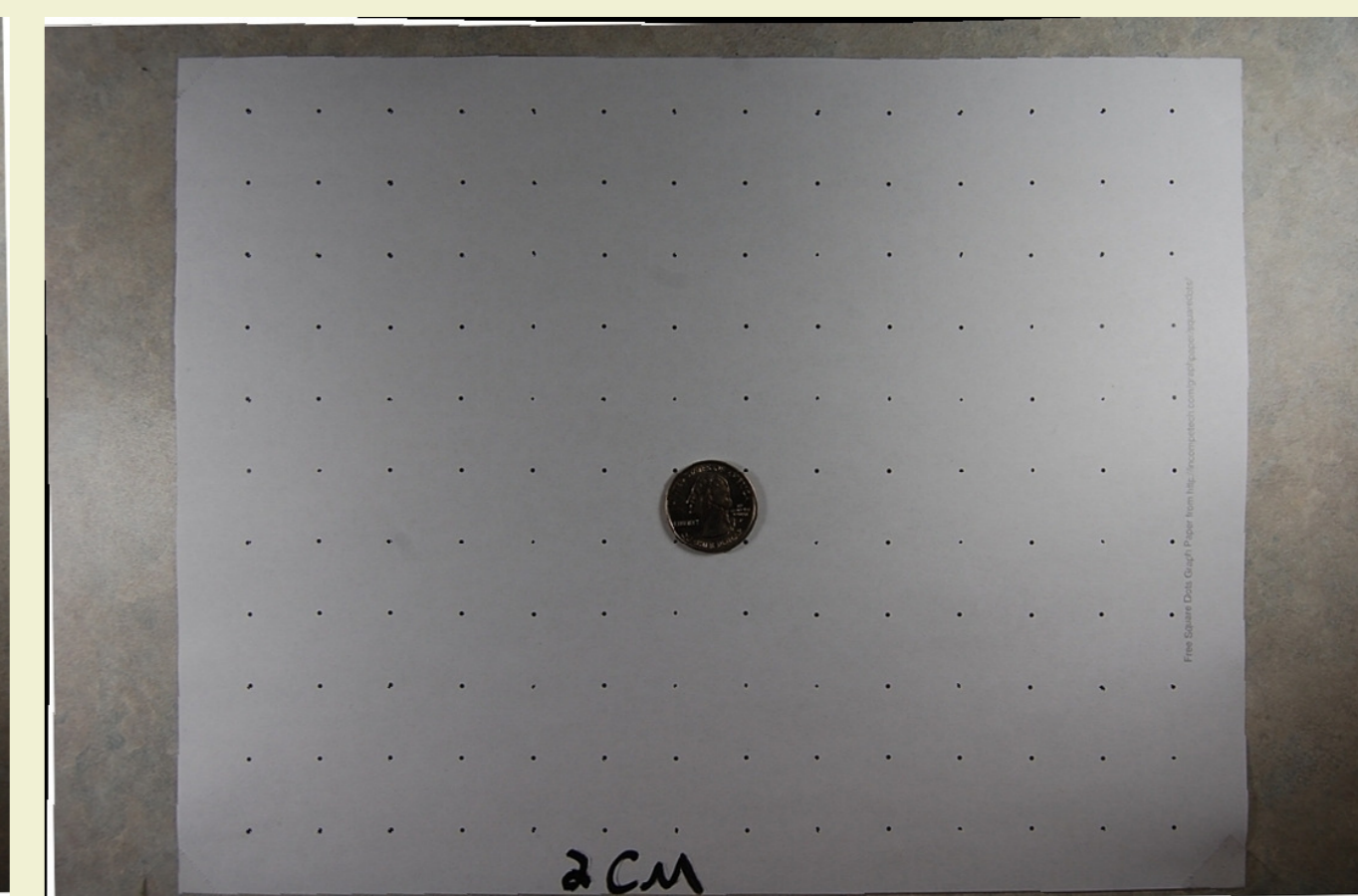
(c) Distorted Image   (d) Corrected Image

Methods

Using thin plate splines, the "distorted" image coordinates can be transformed to the "flawless" coordinates, and the in-between values interpolated, through the mapping $(x, y) \rightarrow (x', y') = (x + f(x, y), y + f(x, y))$ (Bookstein 1989) Software was constructed to apply this method of distortion correction. Java(tm) was the programming language of choice, the Swing(tm) API was used to create the GUI, and the Apache(tm) Math Commons Linear Algebra package was used for all matrix operations. Regular reference grids were generated using the Incompetech(tm) graph paper generator. Points on these grids were used as control points to perform the TPS transformation. Photographs were taken using a Nikon(tm) D40 DSLR camera, with an ED-II Nikkor(tm) lens. The camera was mounted on a Camstand(tm) copy stand, and placed as close to the test page as possible (approximately 1.5 feet). The stand rested on a level table and the camera was leveled before each photograph was taken. To test the authors ability to accurately place landmarks on an image with a mouse, the original 12x14 dot matrix JPG image was digitized and compared to the actual locations of the landmarks. Pre-correction error was measured simply by measuring the distance between the selected distorted points, and their ideal counterparts. Because thin plate splines will always map the control points directly to their target points, a different approach was needed to test the post-correction error. In a 12x13 grid, only landmarks on every other row or column were digitized. The correction was then performed, and the error was measured at the in-between points.

Results

The digitization error test revealed that the author was capable of correctly labeling identifying landmark points with a mean error of 0.23mm, and a maximum error of 0.67mm in an image where each pixel corresponded to 1/3mm of length, and each point had a diameter of 1mm. Prior to correction, the mean error in the image was 1.98mm, with a maximum of 3.61mm. After correction, the mean error in the image was 0.41mm, with a maximum of 0.75mm. A (scaled) displacement vector plot of the 6x7 test image clearly indicates the presence of radially asymmetric distortion.

Discussion

Although it is clear that the software functioned well in correcting lens distortion, it is not entirely clear how significant that error is in the first place. Further analysis must be done to determine how asymmetric lens distortion may effect results, what effect the lens used has on the nature of that distortion, and how the density of control points effects the final error.

Acknowledgments