Bayesian Sampling Methods for Population Genetics

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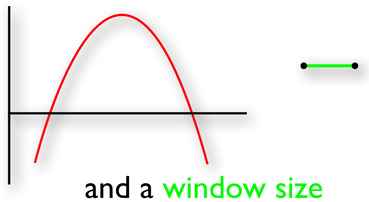
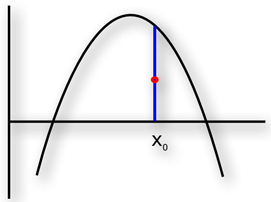
Slice Sampling

as Done by Neil 2003

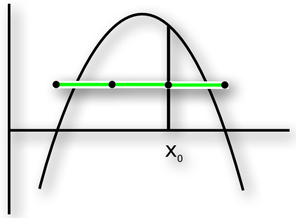
Start with a function to evaluate



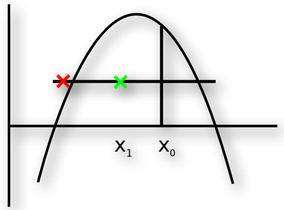
and a window size



Evaluate the function and choose a random height



Step out until outside of function is reached to create slice

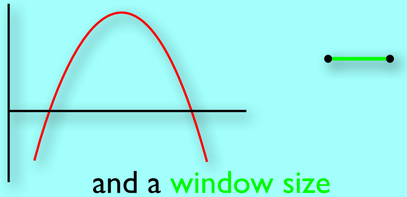


New Point is the first random uniform inside this "Slice"

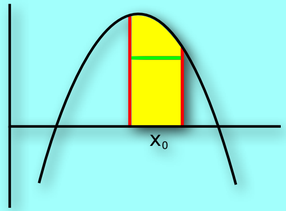
Metropolis Hastings

sliding window proposal

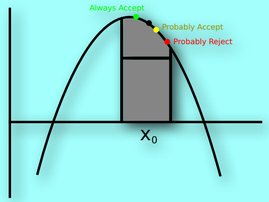
Start with a function to evaluate



and a window size



Choose a new point within this window

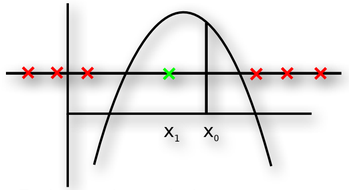


Evaluate this new point, and accept or reject it

$$\text{Probability of acceptance} = \min\left(1, \frac{P(X')}{P(X_0)}\right)$$

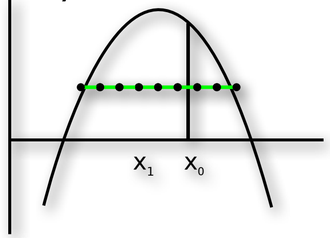
Adaptive Slice Sampling

We noticed that the efficiency of this algorithm was dependent on window size



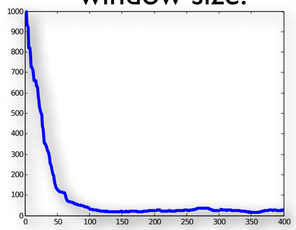
Too large window size leads to many rejections

Too big or too small a window leads to too many function evaluations



Two small a window size requires many steps to reach outside the function

Solution is to dynamically adjust window size.



Iteration vs Stick size

Convergence is quick

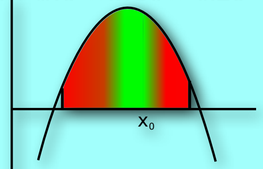
1:41
Adaptive

2:09
non-Adaptive

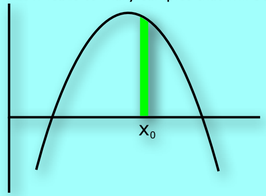
Result is quicker run time

Adaptive Metropolis Hastings

The optimal acceptance rejection ratio is 0.44, depends on the window size



A large window leads to many samples that will be rejected



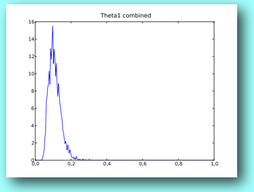
New samples aren't much different from old samples

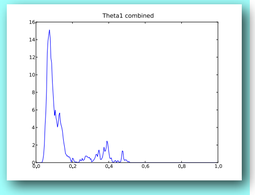
too small or too large a window results in poor sampling

$$\beta = \left(\frac{1}{\alpha}\right)^{-\frac{R-1}{R}}$$

Rejection → multiply window by beta
Acceptance → multiply window by alpha
R → desired acceptance/rejection ratio

We can choose our acceptance rejection ratio(must do this during burn in)



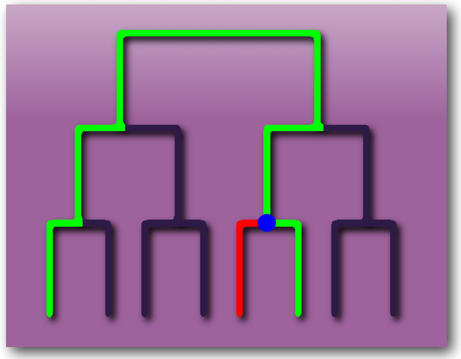


The adaptive algorithm(left) shows better mixing

Adaptive vs. Non adaptive

Future Research

Slice Sampling Trees



Slice sampling and adaptive MH on trees may be beneficial. Here I show one way to slice sample a tree. The red branch can be moved anywhere along the green line to reduce this type of sampling to one dimension at a time

Multi-Dimensional

Currently sampling is done one parameter at a time. We know that these parameters are correlated

Multi dimensional slice sampling from *Neil, 2003*

$$Q_n(x,\cdot) = (1-\beta)N(x,(2.38)^2\Sigma_n/d) + \beta N(x,(0.1)^2 I_d/d),$$

Equation for adaptive multidimensional MH from *Roberts and Rosenthal 2009*

References

Roberts GO and Rosenthal JS (2009) Examples of adaptive MCMC *Journal of Computational and Graphical Statistics* Vol 18(2) 349-367

Neil RM (2001) Slice Sampling *The Annals of Statistics* Vol 31(3) 705-767